BURNHAM–MOORES
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Antifragility of Real Estate Investments in a World of Fat-Tailed Risk*

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† Guy Tcheau
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Abstract

Investors have sought to add real estate to their multi-asset portfolios due to the lower volatility, higher component of total return from income, diversification and tangible nature associated with real estate relative to other assets generally. Real estate is often seen as defensive in this regard. Exogenous shocks or Black Swan events, such as Covid-19, are by definition, 'unknowable' with respect to occurrence and consequence and therefore susceptible to the limitations of statistical models, a priori. This paper examines real estate investing, not just from whether it is defensive, but whether it has antifragility characteristics. Antifragility refers to an investment that is not only robust to exogenous shocks but benefits from such shocks. We show from first principles and from empirical data that real estate has antifragility and warrants higher allocations to multi-asset portfolios for this reason.

Introduction

1.1 Real Estate investments provide several key benefits when added to a multi-asset portfolio. These benefits stem from real estate's inherently low volatility and low correlation vis-a-vis asset classes such as stocks and bonds, as well as inflation hedging potential[1]. In the traditional Markowitz modern portfolio theory construct, real estate improves the risk / return profile by enhancing the mean-variance efficient frontier in a multi-asset portfolio[2]. The diversification benefits from private real estate benefit portfolios in both down and up markets[3]. The high proportion of total return attributable to current income versus appreciation coupled with contractual leases from tenants that undergird the income yield adds defensiveness to the overall portfolio.

1.2 One might argue that real estate returns, risk adjusted, i.e. for volatility, have proven superior to stocks and bonds. The chart below shows real estate having the highest Sharpe Ratio vis-à-vis the other asset classes. The Sharpe Ratio is aimed at measuring how well a portfolio's return compensates the investor for risk. It is the excess return of an investment portfolio over the risk-free rate divided by standard deviation of the returns. Higher returns are not necessarily superior, if it is accompanied by higher variability in the returns. It is the returns per unit of risk that the Sharpe Ratio measures. The higher the Sharpe Ratio the better the returns relative to risk, measured in this fashion.

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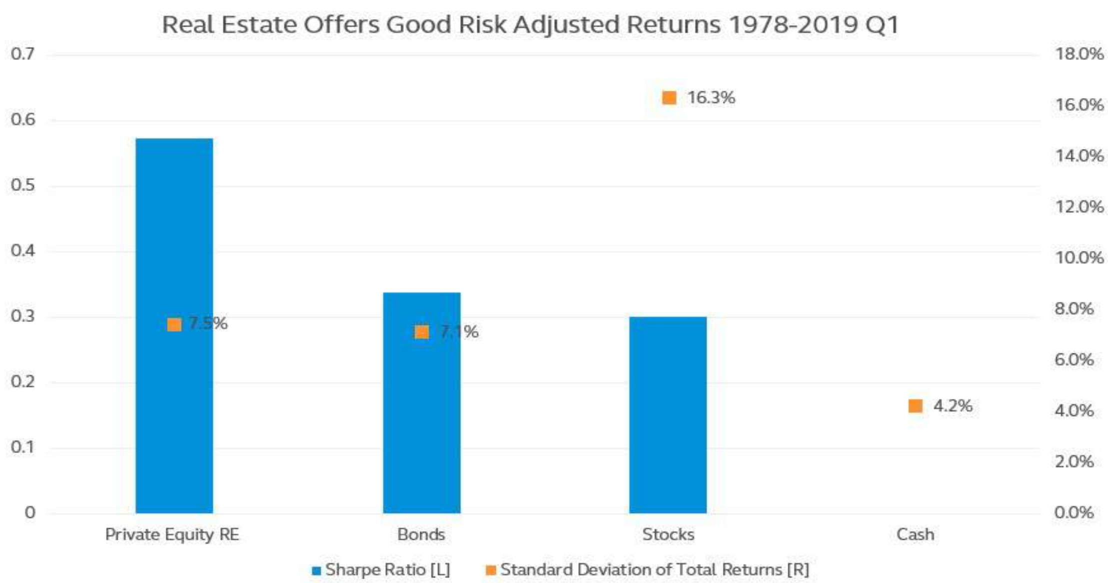


Sources: NCREIF, Barclays, Standard & Poors, Moody's Analytics

1.3 In addition, real estate investments can be accessed via four quadrants which provides a menu of risk and return alternatives for the investor.

EQUITY VS DEBT / **PUBLIC VS PRIVATE**

	INCOME & APPRECIATION	YIELD & DOWNSIDE PROTECTION
LONGER EXECUTION & LOWER VOLATILITY	PRIVATE EQUITY : Core, Value-Add Opportunistic	PRIVATE DEBT : Senior mortages, Mezzanine loans Construction loans Participating loans
LIQUIDITY & HIGHER VOLATILITY	PUBLIC EQUITY : Listed developers Public REITs Preferred Securities	PUBLIC DEBT : AAA - Non-investment grade CMBS Interest only CMBS REIT unsecured debt

These four quadrants are obtained by combinations along two dimensions, namely public vs private and equity vs debt as shown in the figure above.

1.4 This paper examines the attractiveness of real estate investing from another vantage point called ‘robustness and anti-fragility’. This thesis flows primarily from the published work of Nassim Taleb on the subject of mathematical finance, uncertainty and randomness. By coincidence, the fourth quadrant is also referred to by Taleb in his book the Black Swan. However, the two dimensions in Taleb’s model are simple payoffs vs complex payoffs and thin-tailed vs fat-tailed distributions of outcomes. The fourth quadrant thus refers to financial outcomes that cannot be modeled or statistically quantified with sufficient robustness to permit qualified decision making. This quadrant is the domain of Black Swans, where investors are susceptible to the limitations of statistical models as applied to real world finance because of what Taleb calls the ‘unknown unknowns’[4].

1.5 Institutional investors such as pension plans typically have an allocation of their multi-asset portfolios to real estate. Prior to the current Covid-19 dislocation, real estate allocations for pension plans were averaging about 10.4% versus target allocations of 11.4% based on surveys by leading real estate associations (PREA, INREV and ANREV)[10]. Due to the decline in the stock market, these real estate allocations are likely to be near or overweight the targets due to the denominator effect, i.e., the fall in value of the overall portfolio relative to real estate value adjustments. Given the antifragile nature of real estate it can be argued that target allocations to real estate ought to be increased.

2. Probability Distribution

2.1 Let us examine the two ways to view the real world; the first of which Taleb calls 'mediocristan'[5]. In this view, outcomes are fairly predictable and stable. Within a large sample size, no single outlier outcome significantly impacts the aggregate total. Mediocristan is characterized by event probability distributions that tends towards the traditional thin-tailed normalized distribution. In other words, there exists a mild randomness which is reasonably easy to parametrize and model.

2.2 The other view of the world, which is rarely used in finance, is called 'extremistan'[5]. Here, one outlier observation can materially distort the aggregate. Randomness is not mild but severe. Rather than a normalized bell-shaped probability distribution we have instead a fat or fat-tailed curve. Outcome distributions are skewed or have high kurtosis[6].

2.3 It follows that that given the two sets of outcomes i.e. mediocristan or extremistan, we will have two distinct categories of investment decisions depending on whether mild or severe randomness is in force. Mediocristan based decisions are generally simple and binary i.e. either yes or no; true or false. Decisions are based solely on probability of events and are not impacted by the consequence of the outcome. Extremistan based decisions are much more involved. Both the probability of an event and the consequence of the outcome have to be considered in tandem.

EVENT PROBABILITY	SIMPLE PAYOFF	COMPLEX PAYOFF
THIN TAILED, NORMAL DISTRIBUTION OF EVENT FREQUENCY	High Certainty & Low Consequence Highly Predictable Outcomes Robust to Black Swans	High certainty & Large Consequence Somewhat Predictable Outcomes Fairly robust to Black Swans
THICK TAILED OR ASSYMETRIC DISTRIBUTION OF EVENT FREQUENCY	Less Certainty & Low Consequence Somewhat Predictable Outcome Fairly Robust to Black Swans	Less Certainty & Large Consequence Unpredictable Potentially Big Outcomes Extreme Fragility to Black Swans

4th Quadrant

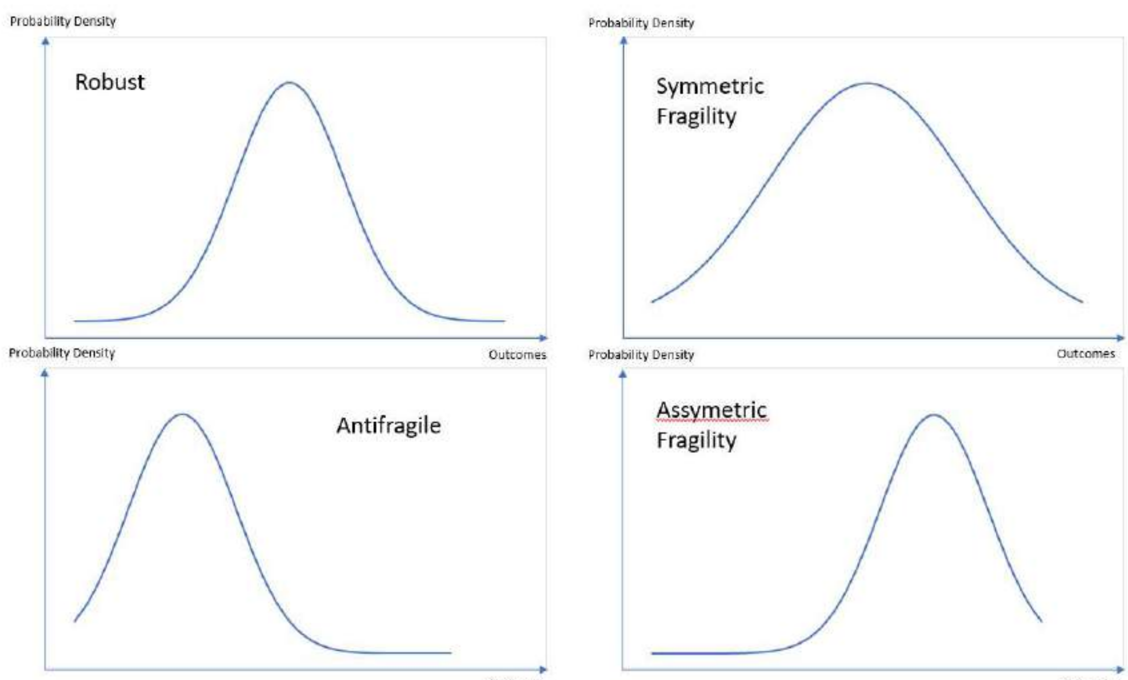
2.4 Traditional financial models based on mean variance assumes normalized Gaussian distributions and thin tails[6&7]. The normalized distribution clusters outcomes symmetrically around the mean where the probability of an outcome being 2 standard deviations away from the mean is under 5%. The probability falls to 0.3% if the outcome is 3 standard deviations away from the mean. Thin tails refer to very low probability of outcomes occurring outside of 3 standard deviations or three-sigma from the mean. However, the reality is that the measures of averages and variances that are relied on can be very misleading because distributions in the real world are more asymmetric than commonly assumed. Furthermore, so called thin tailed outliers occur with higher frequency then implied by normalized distributions. We call probability distributions fat tailed when the outcomes that deviate from the mean by 3 or more standard deviations are materially higher than implied in a normalized distribution.

2.5 As a consequence, portfolio construction based on mean variance modeling, while generally effective in 3 of the 4 outcome scenarios, become fraught with risks when applied to the 4th quadrant due to the high degree of fragility to Black Swan events. A casual observation of the Global Financial Crisis, Covid-19 and other systemic shocks bears witness to the impact of outcomes in the 4th quadrant.

2.6 With the benefit of hindsight, financial market participants adjust quantitative models to account for the latest systemic shock followed up by stress tests to derive comfort that portfolios will be immunized from repeat debacles. However, the problem with ‘unknown unknowns’ is that it falls outside the bounds of robust statistics and quantitative models, prima facie.

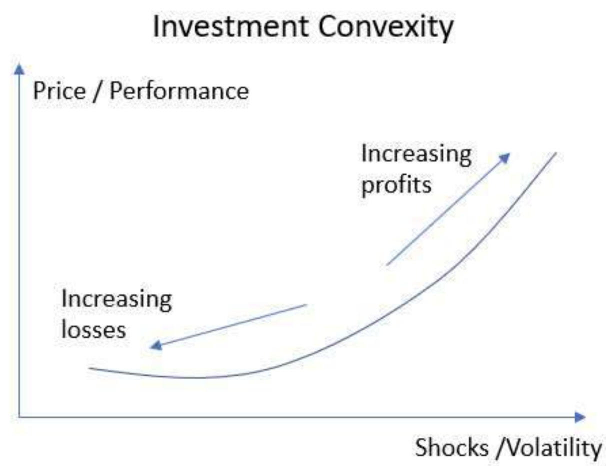
3. Antifragility

3.1 There are essentially four outcomes as shown below[8]. Robust outcomes are where outlier events have low probability and have small consequence, either positive or negative. Investment performances are fairly predictable since outcomes are not materially impacted by shocks. However, there are two types of fragile outcomes: symmetric and asymmetric, which are both descriptors of downside scenarios. Both give rise to more negative impacts since the tails are fat. Asymmetric tails to the left means that the investment is more fragile to negative outcomes than to positive upsides. Antifragile[9] investments, on the other hand, are asymmetric to the right. Here the fat tail to the upside means outliers tend to be positive with large payoffs. In short, robust investments are agnostic to volatility whereas antifragile investments benefit from dislocations. Fragility implies that investments tend to go very badly with volatility.



3.2 What exactly are antifragile investments? These are investments that benefit from volatility. Mathematically this is referred to as convexity.

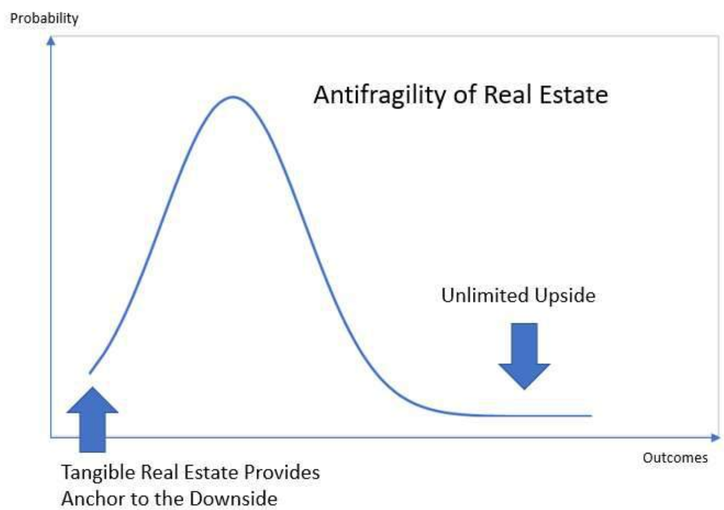
Convexity under Taleb's model is different from convexity that is used for bond investments. For bonds, convexity measures the change in the price of the bond in relation to changes in the level of interest rates. As the diagram below illustrates, the higher the volatility in the events under Taleb's model the greater the payoff with convexity[12]. Conversely, greater volatility results in greater losses for concavity.



3.3 With these 4 outcomes in mind, Taleb advocates the majority of a portfolio be concentrated in robust investments with a small portion in antifragile investments. Fragile investments are to be shunned altogether. This barbell strategy seeks to resolve the problem with extremistan and the 4th quadrant of Black Swan event.

4. Antifragility of Private Equity Real Estate

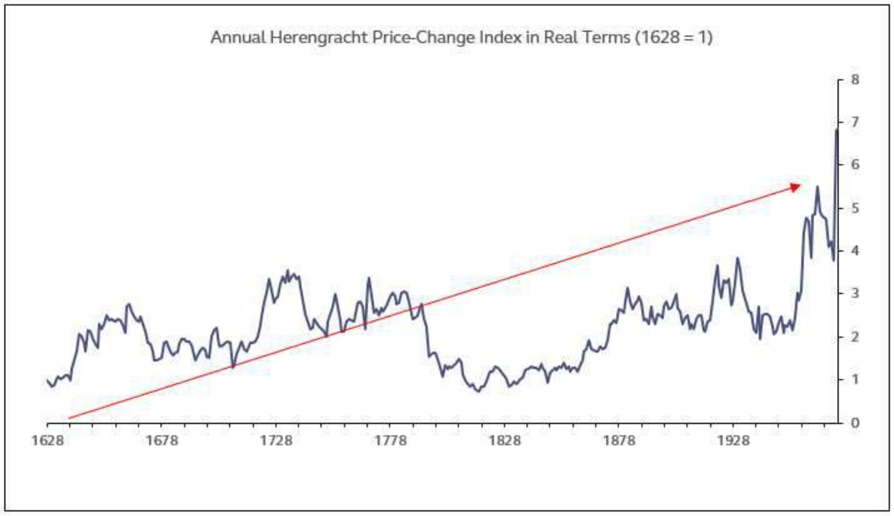
4.1 We believe that private equity real estate exhibits antifragile characteristics because of its asymmetric return profile that is skewed right. While these antifragile characteristics may have some degree of manifestation in real estate debt or publicly traded REITs, it is most pronounced and observable in private equity real estate performance. We focus on the private equity real estate exclusively when examining antifragility and convexity.



4.2 With public equities or corporate bonds for example, in Black Swan events, the value of certain securities might tend towards zero. The reason is that the underlying businesses can become unviable due to shocks. Take for example Lehman Brothers or Bear Stearns during the Global Financial Crisis. These firms were no longer able to be remain operational. With Covid-19 it remains to be seen whether certain cruise ship operators will remain in business, for example. However, with real estate there is real tangible property that underpins the investment. Real property has been a repository of value for many millennia. Real estate provides a non-trivial anchor to the downside which is largely overlooked in traditional financial models. Whereas the upside is technically unlimited, the downside has a floor. This makes the return distribution asymmetric to the right.

4.3 The empirical data on real estate price changes in real terms over very long time period of nearly 400 years (See the case study on housing prices in Netherlands below) supports asymmetry to the upside and finite downside. Despite major Black Swan events such as the Black Plague, Napoleanic Wars, both World Wars and the Great Depression, real estate showed resiliency to the downside (floors provided by tangible real property value) plus fat-tailed upside. Note also, that this period included the Spanish Flu of 1918 which reportedly infected over 500 million people with a death toll between 40 million to 100 million.

Housing Data Netherlands



1635 – Black Plague
1672 – Anglo-French War
1701 – Spanish Succession War
1794 – Napoleon
1914 – World War 1
1930 – Great Depression
1940 – World War 2
1970 – Oil Shock

Source: Eichholtz & Geltner (2004)

4.4 The next question is whether real estate exhibits convexity. On a relative return basis, as compared to public traded stocks or corporate bonds, real estate fares relatively well during periods of systemic distress. Therefore, we can infer that convexity exists in the relative sense. Yet, it is true nonetheless, that in the short term, Black Swan events do hurt real estate values. Therefore, real estate appears to exhibit concavity in the short term. However, when viewed over a longer-term horizon, rather than focusing on spot prices, we see evidence that real estate losses halt due to its intrinsic real value and then, not just recover, but move to the upside.

4.5 The reason is that real estate values are a function of rents paid by property occupiers. Rental rates are derived from supply and demand forces for both occupiers and suppliers of real estate. Following Black Swan events, the supply of capital for construction of new properties becomes highly curtailed. This chokes off new supply. However, the demand for real estate which is a derivative of population and employment increases in aggregate over the long term. This natural phenomena for demand results in a rightward shift in the demand curve while the supply curve either shifts left (obsolescence) or remains relatively constant in the aftermath of shocks. The combined effect leads to equilibrium pricing (rents) moving upwards. As rents rise so will real estate values, which discounts the stream of rental payments over the investment horizon. We posit that these forces result in convexity for real estate investors.

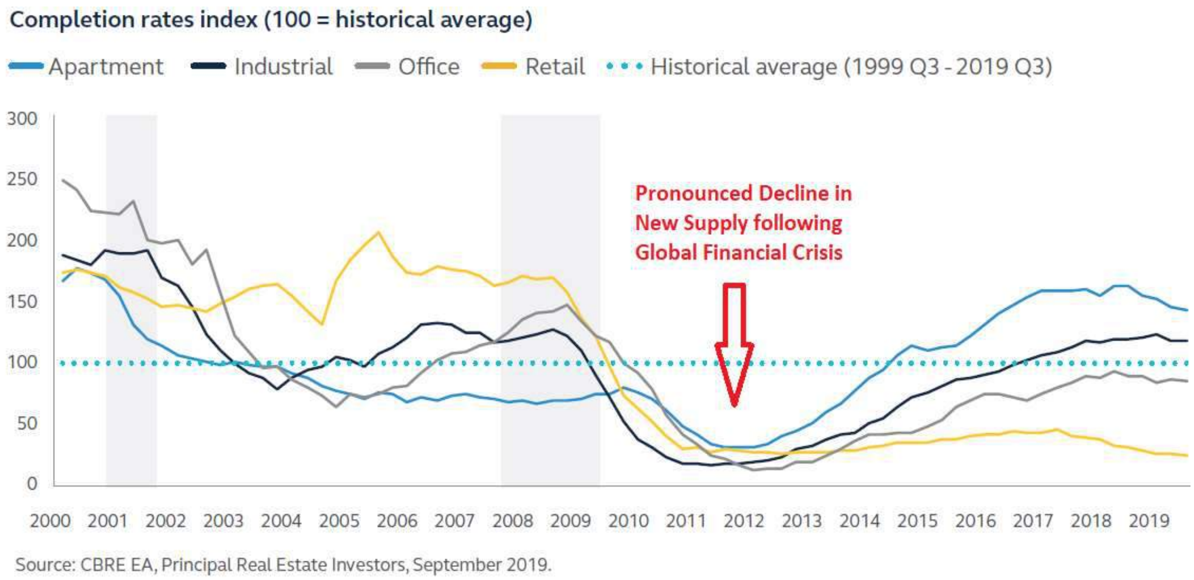
5. Convexity of Real Estate from First Principles

5.1 To see the causal effects for convexity we compare the dynamics of real estate demand and supply with that of traditional businesses that underpin public equities and bonds. In the case of businesses in general, the demand side of the supply and demand equilibrium has the larger impact on success and viability. Businesses cater to their customers but cannot control the state of the customers and their demand at any point in time. We characterize businesses in the aggregate as predominantly 'exogenous'.

5.2 Real estate also caters to their customers who in this instance are the property occupiers. Demand is a function of the number of occupiers which in turn is a derivative of general population. As noted earlier, general population in aggregate increases over the long run. Therefore, it follows that occupier demand for real estate has a natural rightward shift; that is after smoothing out shorter term cyclical impacts of macroeconomic dislocations or localized sociopolitical perturbations. The more potent force in the demand supply equilibrium of real estate is the supply side. In other words, boom bust cycles in real estate are driven more by overbuilding of real estate than by demand side shifts; ceteris paribus.

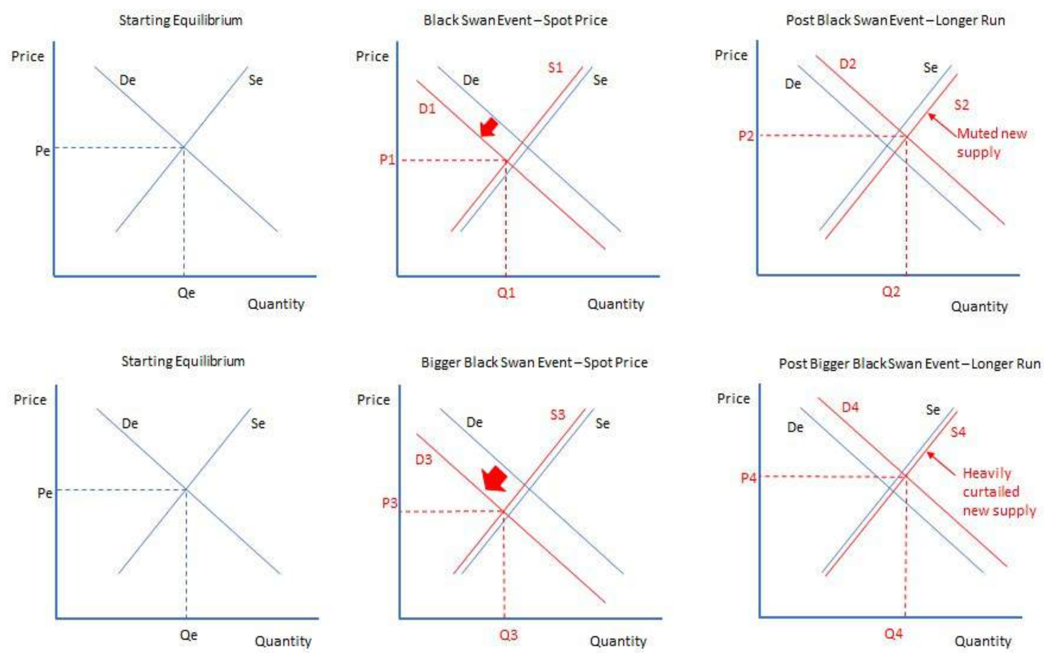
5.3 We are, of course, speaking about real estate in aggregate as segments of real estate by type or geography certainly do react to demand forces. Retail for example is greatly impacted by e-commerce, which in turn has decimated many bricks and mortar retailers, who are the occupiers of retail malls. The destruction of the demand base for retail has materially impacted retail mall rents and hence values, while benefitting logistics and industrial properties. However, across real estate in aggregate which also includes multifamily apartments, office, industrial, hospitality, housing, farmland, etc., we can see that demand exhibits less variability with a gradual right shift due to population increase. It follows then that supply is the principal driver of equilibrium in real estate demand and supply interaction. We characterize real estate as predominantly 'endogenous'.

5.4 Following the Global Financial Crisis, we can see a dramatic drop off in the level of new supply across real estate. This stems from fear and risk-off mentality coupled with anti-risk-taking measures imposed by policy makers which act to curtail real estate developers' access to capital. Paradoxically the more severe the shock i.e. greater volatility, the more severe the resulting curtailment of new supply. This is precisely the necessary force needed in a predominantly endogenously driven demand and supply market dynamic to produce the upwards pricing equilibria in response to volatility.



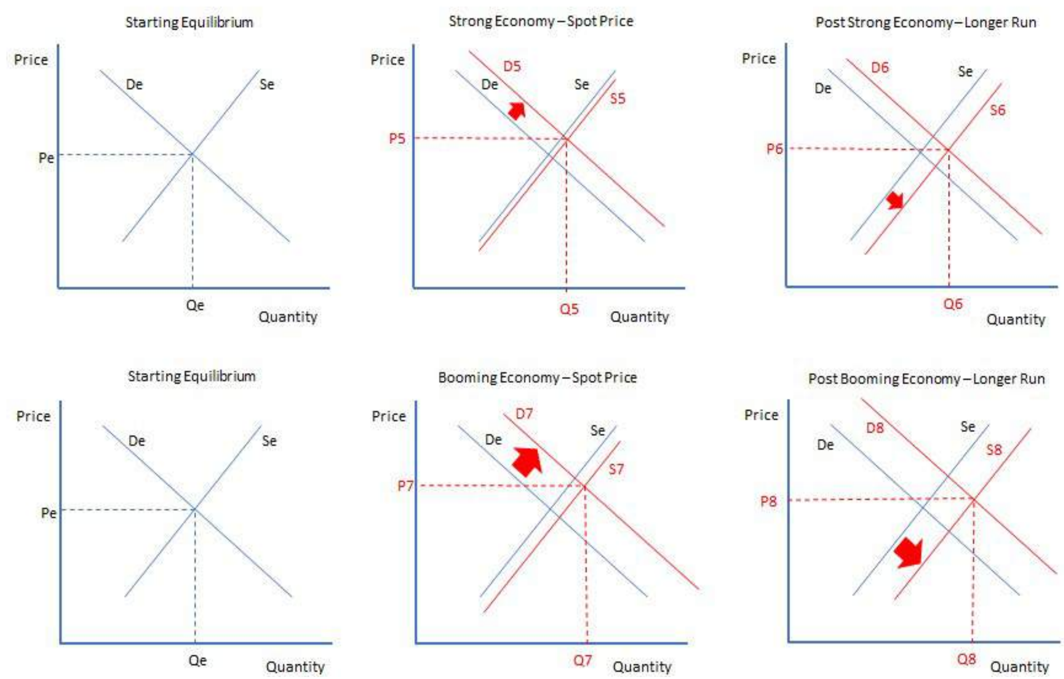
5.5 We examine the demand supply equilibria for real estate in the short and long run under conditions of increasing volatility (Black Swans) and decreasing volatility (strong macroeconomy). Starting with equilibrium between demand and supply with $\text{Price} = \text{Pe}$ and $\text{Quantity} = \text{Qe}$ we first examine the impact of a Black Swan even on the spot price. We see that demand falls off materially to $D1$ due to the shock to aggregate demand. There is a drop off in Supply to $S1$ which is small relative to the fall in demand. In real estate, supply decreases due to obsolescence or the withdrawal of marginal properties from the market. However, unlike manufacturing where production lines can be halted, real estate supply is sticky as it is essentially a fixed asset. At the new spot equilibrium, price falls to $P1 < Pe$. New supply of real estate in the longer run post Black Swan events becomes muted as providers of capital for real estate development become significantly risk averse and policy makers institute anti-risk taking measures. As a result, the supply response $S2$ is relatively small as demand recovers and increases with the tailwind of general population growth. Price equilibrium moves to $P2 > P1$ and $P2 > Pe$.

5.6 Paradoxically, the more severe the Black Swan event the stronger the curtailment of new supply in the longer run. In a bigger Black Swan dislocation, demand falls more dramatically to D3 and price to P3<Pe. As the shock is more severe, price P3<P1 in the short run. The severity of the shock generates a stronger and more lasting reaction in the new supply of properties as providers of capital for development become even more risk-averse and regulations become more draconian against development funding. As demand recovers to D4 it is the endogenous force of heavily curtailed new supply that drive equilibrium to P4>Pe. Note P4>P2, meaning the long run equilibrium achieves higher pricing the more severe the Black Swan event.

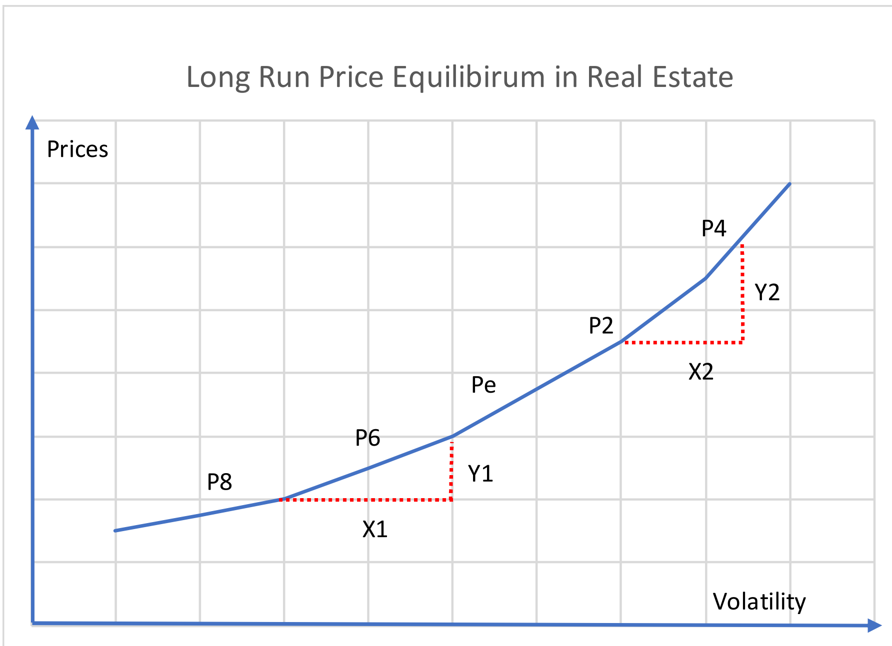


5.7 Now, let us examine demand and supply equilibria of real estate under conditions of economic growth. Here, we have no outlier events and volatility of events is declining. In the short run, aggregate demand increases to D5 and supply increases to S5. Since new supply in real estate is sticky in the short run D5 outweighs S5 and the equilibrium price moves to P5>Pe. Since property development requires time to source capital, obtain government approvals, mobilize and construct there is a lag to new supply. The impact of exuberant macroeconomic conditions leads to ample supply of capital for property development. This results in a spike in new supply S6 which acts to swamp the growth in demand to D6. As noted earlier, real estate equilibrium is predominantly a function of endogenous forces. We find $P6<P5$. If this coincides with adverse macroeconomic conditions impacting demand, we could have $P6<Pe$. This is the traditional real estate boom bust cycle.

5.8 If instead of stable macroeconomic conditions, we have a booming economy, then demand moves to $D7$. New supply of real estate increases to $S7$. While $S7$ is larger than $S6$ stickiness in new supply and lag means that the supply response is insufficient. $D7$ acts to swamp $S7$ and price equilibrium moves to $P7>P5$ and $P7>Pe$ in the short run. Interestingly, in the longer run, risk-on prevails and capital for property development becomes plentiful and inexpensive. The new supply response is even more pronounced, and supply moves to $S8>S6$. Although demand $D8$ is increasing it cannot keep pace with the wave of new construction deliveries and price equilibrium adjusts to $P8<P7$. If the timing of this adjustment coincides with a slowdown in the macroeconomy we could have a more severe price correction or collapse where $P8<P6$ and $P8<Pe$.



5.9 The combination of an endogenous demand and supply equilibrium and a natural right shift in demand for real estate that is a derivative of general population growth melds to give us a function of real estate occupier prices (aka rents) relative to the presence or absence of Black Swans. Very simply we can plot systemic shocks versus occupier prices in the long run. Since rents are in effect a proxy for real estate values or prices, we can infer that real estate pricing outcomes will follow occupier rents based on the demand and supply equibria developed above. Convexity is inferred from first principles. What we mean is that in the following chart, the slopes $\frac{Y2}{X2} > \frac{Y1}{X1}$ $i.e. \left.{}^{\partial y}/_{\partial x}\right|_{\{P4\}} \geq \left.{}^{\partial y}/_{\partial x}\right|_{\{P6\}}$.

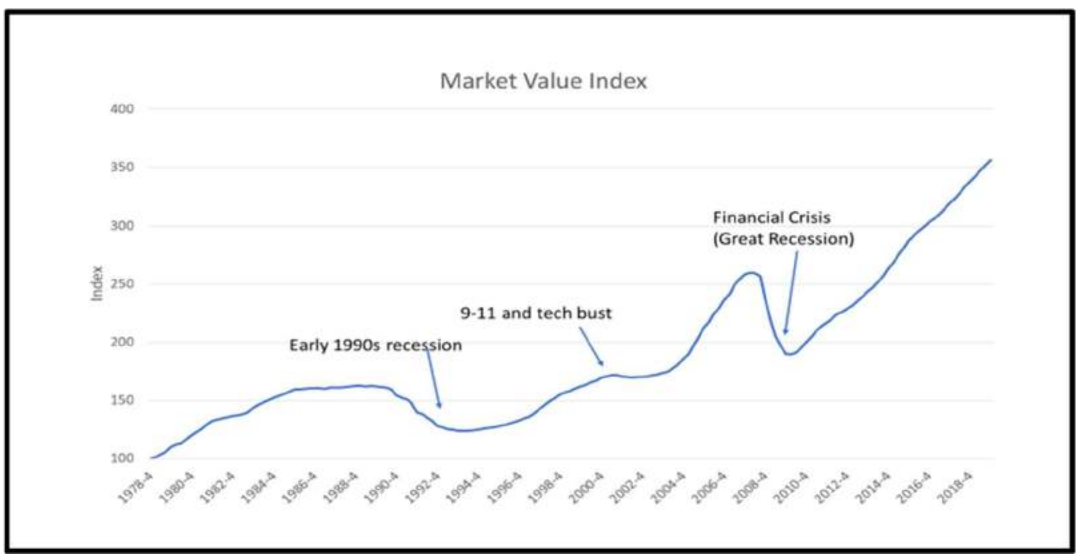


6. Institutional Real Estate Returns Following Downturns

6.1 Our thesis is that the combination of curtailed new supply plus as a result of risk aversion and policy response from Black Swan events plus revived occupier demand recovering post-shock with support from the natural right shift progression derived from general population growth produces gains for real estate that are asymmetric to the right. The bigger the Black Swan event the more acute the ensuing new supply curtailment in real estate in the long run. This then results in more sustained upside in real estate occupier rents and prices.

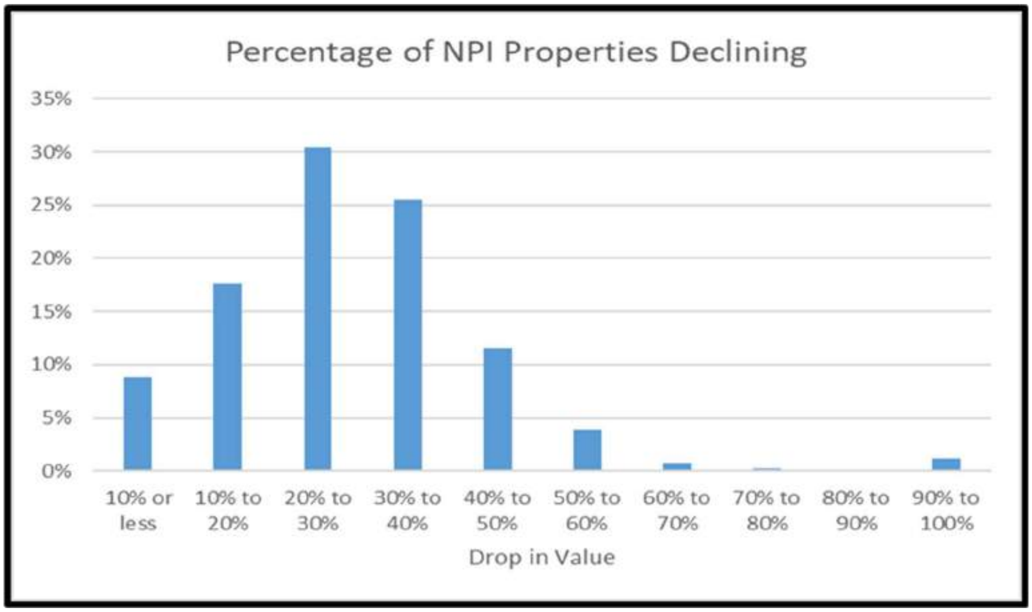
6.2 We now seek to examine the performance of institutional owned commercial real estate under various conditions of macroeconomic shocks. Institutional real estate is the primary investible segment of real estate for pension funds. The National Council of Real Estate Investment Fiduciaries (NCREIF) index is a widely used benchmark that tracks income and appreciation of institutionally owned commercial real estate (CRE). The quarterly data series starting in 1978 includes the performance of over 40,000 office, industrial, retail, multifamily and hotel properties plus other specialized property types such as self-storage and senior housing.

6.3 The following chart shows how the market value of NCREIF properties has changed over the past 42 years[10]. There is a parallel, albeit on a shorter time series, with the 400 years of data on housing prices in the Netherlands that we looked at earlier. We note that property values have increased over time despite intervening systemic shocks as evidenced by the rising market value index (MVI) over time. MVI is an equal weighted index which seeks to reduce the impact of value changes of large properties on the aggregate index. We observe that in each recession episode, value falls hit a floor, which is supported by the tangible value of real estate, and then rebound thereafter.



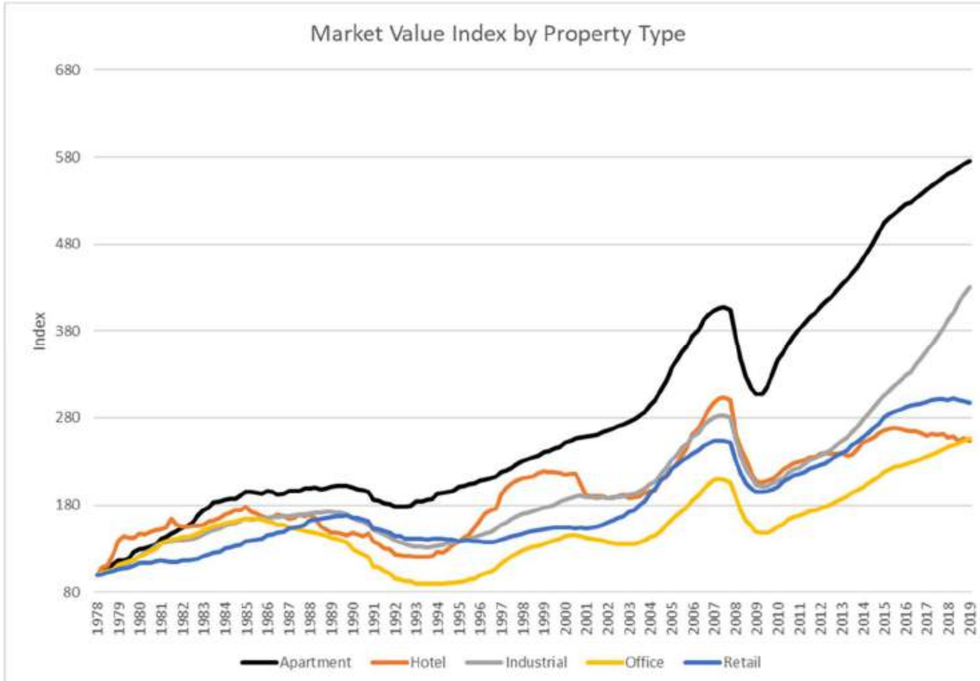
Source: NCREIF & Jeffrey D. Fisher, Ph.D.

6.4 We have asserted that the tangible and intrinsic value of real estate provides a floor to the downside during period of declines. This makes the return distribution asymmetric and skewed right. Examining value declines in commercial property values as a result of the Global Financial Crisis lends light for this thesis. Looking at property value declines, peak to trough, using NCREIF data, we stratify the universe by percentage of declines. The following chart shows that the bulk of the declines ranged from 20%-40%[10]. Absent outliers, 50% value falls were the lower bound.



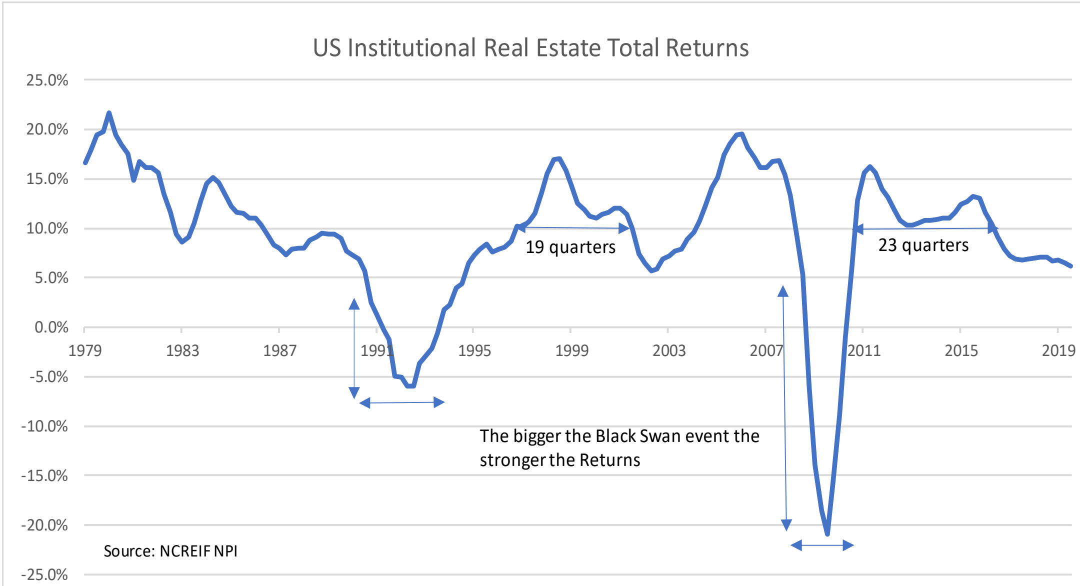
Source: NCREIF & Jeffrey D. Fisher, Ph.D.

6.5 We asserted earlier that while real estate responds in aggregate to shocks, individual sectors of real estate respond differently and may in fact move more autonomously than the aggregate. The following chart shows the MVI index broken out by the five principal sectors, namely, apartments, hotel, industrial, office and retail[10]. We can see variability in value changes over time between each sector.



Source: NCREIF & Jeffrey D. Fisher, Ph.D.

6.6 The following chart shows total returns of institutional real estate from the NCREIF (NPI index). Unlike the market value index, total returns in NPI include income returns in addition to appreciation. We note that the 1991 Credit Crunch and Savings and Loans Crisis resulted in a real estate collapse. Total returns, meaning depreciation in property prices after factoring net income from rents, reached -13.4% in 1992. This was followed by 19 quarters of above 10% annualized total returns i.e. appreciation in property prices plus net income from rental revenues. The Global Financial Crisis resulted in a more severe real estate collapse which hit -26.7% in total annual return in 2009. This was followed by 23 quarters of above 10% annualized returns in real estate. The anecdotal data suggests the larger the dislocation the stronger the ensuing performance in the long run.

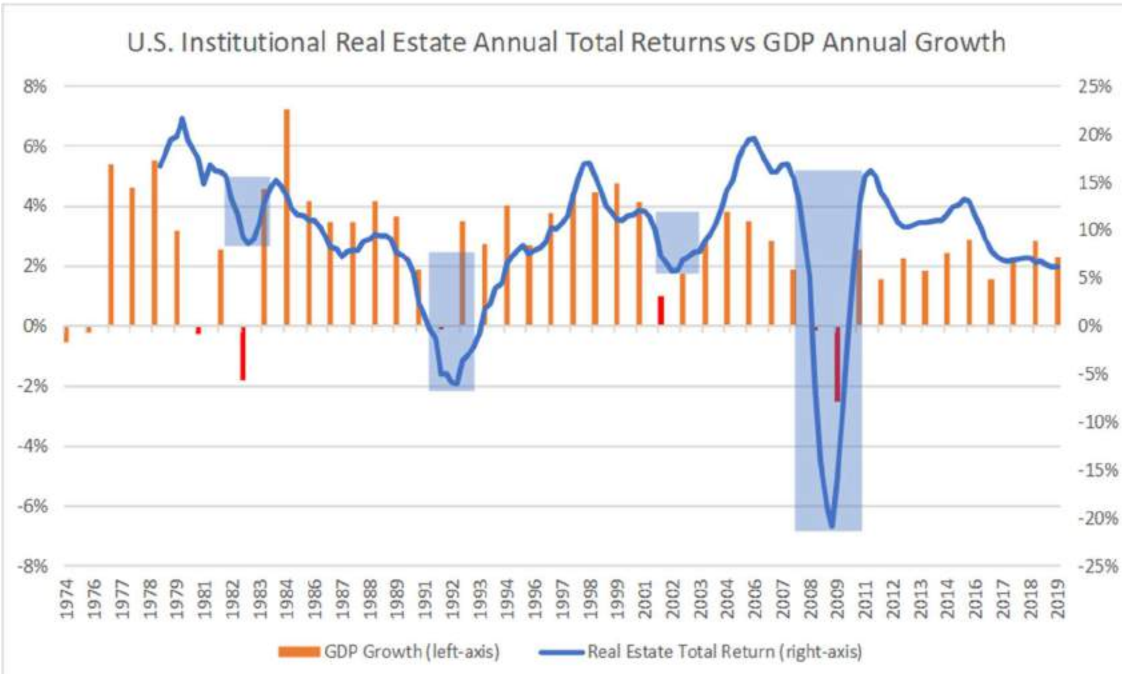


7. Empirical Data for Real Estate Convexity

7.1 Thus far we have inferred convexity based on first principles effected by means of curtailment of new construction following market shocks. The slower it takes for new supply to return the longer rental pricing power favors owners. We also saw anecdotally that the length of real estate outperformance corresponds to the depth of the real estate decline following systemic shocks. We now seek to examine convexity empirically.

7.2 A systemic shock refer to the widespread adverse impact on the financial system stemming from an unforeseen cause. The cause can be endogenous to financial market meaning that the shock was a result of a negative event triggered by a problem from within the financial industry. Or the cause may be exogenous where the source of the disturbance emanates from outside the field of finance. Terrorism, war, plagues are examples of exogenous shocks. The size of the adverse impact on real estate performance is a function of the type and severity of the shock. For the purposes of studying real estate's antifragility we are less concerned with source of the shock and the immediacy of the decline in real estate values but we are focused on the length and strength of the recovery of real estate in the years following the shock.

7.3 As a measure for Black Swan events or market shocks we use real GDP declines. We can see that real estate total returns are impacted by external shocks in the following chart. We use annualized total returns from the NCREIF return index on institutionally owned commercial real estate as a measure for real estate performance. We can see visually that each downturn is followed by a sustained period of strong real estate returns.



Source: NCREIF

7.4 To test for convexity we need to plot real estate performance against volatility. We use annual percentage change in real GDP as a proxy for volatility. The bigger the shock the greater the downside to GDP growth. We compare a marked slowdown in GDP growth (which includes an outright recession) with the average level of GDP growth in the prior 2 years. In the case of the 1982 recession we compared only against 1981 as 1980 was also a recession year. In the case of 2001, while GDP growth was slightly positive, the less than 1% growth was a marked fall from the 4%+ growth in the prior 2 years. The 3% divergence in growth represented the effects of a shock which in this case was the 9/11 terror attacks.

7.5 Following each episode of GDP volatility there followed a period of sustained real estate outperformance. We calculated the cumulative total returns using NCREIF performance data in each case. We then adjusted the returns by CPI to obtain inflation adjusted or real returns. We then plotted volatility as measure by GDP declines against the cumulative total real returns from real estate following each shock.

Point	Severity of GDP Shock	Cummulative Real Returns
C (1982)	4.34%	54.72%
A (1991)	2.89%	55.91%
B (2001)	3.44%	56.40%
D (2008-09)	5.04%	80.76%

7.6 Can convexity be demonstrated empirically? In mathematics, based on Jensen's inequality[11], in order for function $f(X)$ to be convex then $f\left\{\frac{(a+b)}{2}\right\} \leq \frac{\{f(a)+f(b)\}}{2}$. We can apply this proof.

7.6.1 $f(A) = 55.91; f(D) = 80.76; where\ A = 2.89\ and\ D = 5.04;$ $then\ \frac{\{f(A)+f(D)\}}{2} = 68.33.\ f\left\{\frac{(A+D)}{2}\right\} = f\{3.96\}$

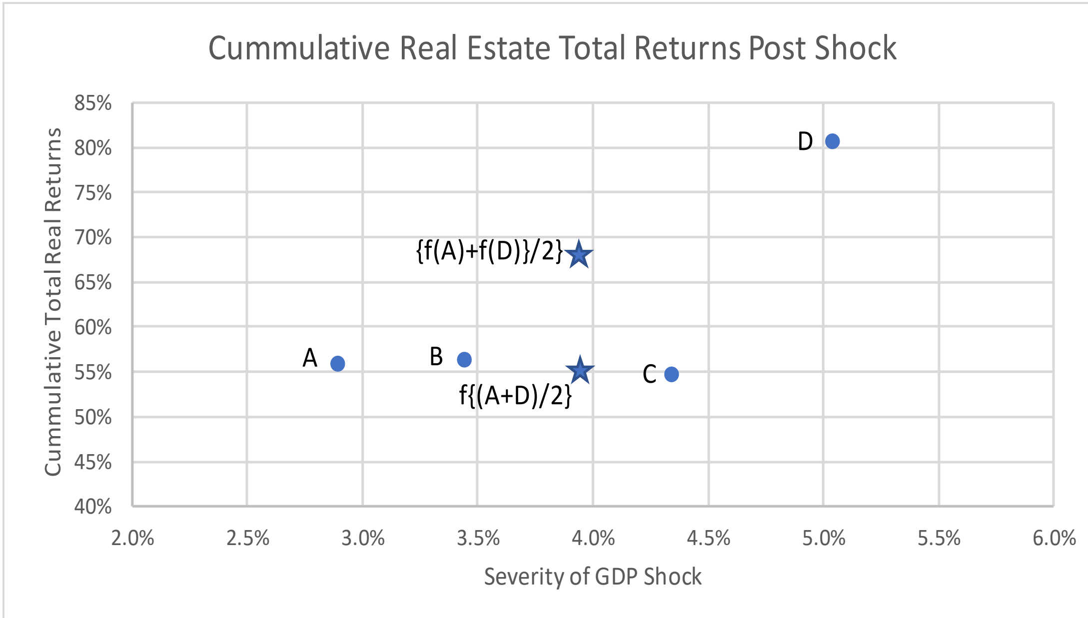
From the chart we visually observe the $f\{3.96\}$ is in between point B and C. The result will be roughly 55 and will not be more than 60. Since $55\ to\ 60 < 68.33$ the proof is valid. The points $\frac{\{f(A)+f(D)\}}{2}\ and\ f\left\{\frac{(A+D)}{2}\right\}$ are indicated in the chart below. Clearly $\frac{\{f(A)+f(D)\}}{2}\ is\ above\ f\left\{\frac{(A+D)}{2}\right\}$.

7.6.2 $f(B) = 56.40; f(D) = 80.76; where\ B = 3.44\ and\ D = 5.04;$ $then\ \frac{\{f(B)+f(D)\}}{2} = 68.57.\ f\left\{\frac{(B+D)}{2}\right\} = f\{4.24\}$

From the chart we visually observe the $f\{4.24\}$ is in between point B and C. The result will be roughly 57 and will not be more than 60. Since $57\ to\ 60 < 68.57$ the proof is valid.

7.6.3 $f(A) = 55.91; f(C) = 54.7; where\ A = 2.89\ and\ C = 4.34;$ $then\ \frac{\{f(A)+f(C)\}}{2} = 55.31.\ f\left\{\frac{(A+C)}{2}\right\} = f\{3.61\}$

From the chart we visually observe the $f\{3.61\}$ is in between point B and C. Since the distance between B and C is fairly short, the result will on the line (more likely under the line) between B and C, which is congruent with line between A and D. $f(3.61\} \leq 55.31$ supporting the Jensen inequality for convexity.



7.7 Another feature of convexity is that the sum of $f(a) + f(b)$ decreases by "smoothing" a and b together. $f(A) + f(D) = 136.7$. Smoothing a and b, we select B and C since both points are inside of A and D. $f(B) + f(C) = 111.1$ which is smaller than $f(A) + f(D)$ as expected, supporting the proof for convexity.

7.8 We approach convexity by attempting linear, superlinear and sublinear curves to fit the empirical distribution found above. We are not seeking a perfect fit but rather the shape of the curve with reasonable accuracy.

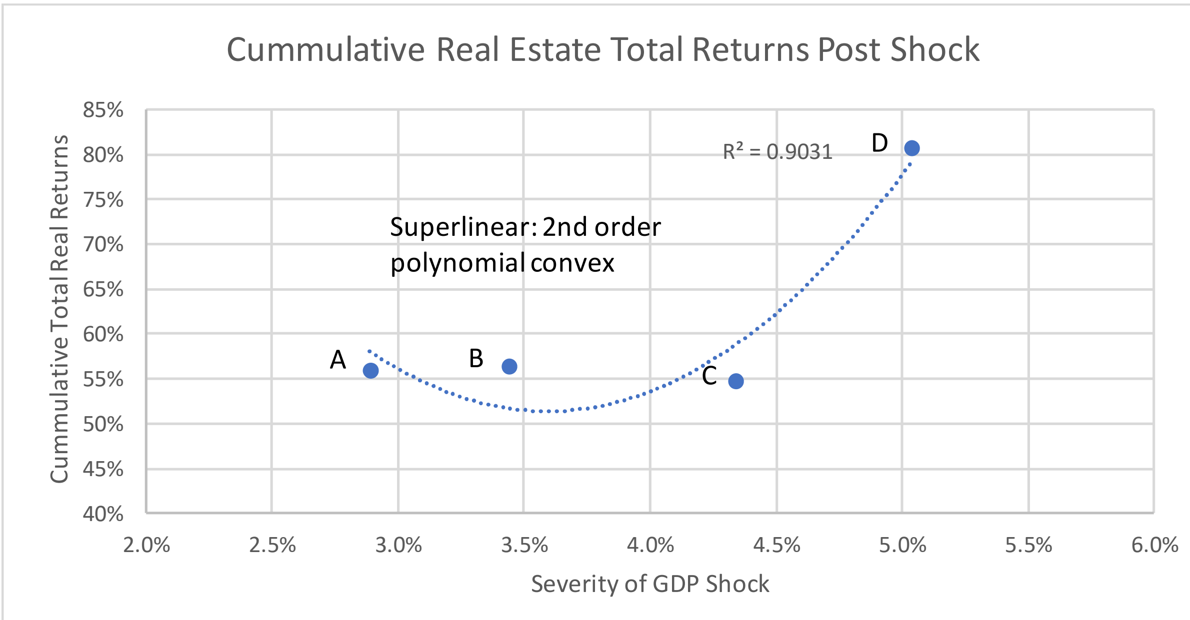
7.9 We started with a second order linear regression as shown in the following chart. The goodness of fit R^2 at 0.9031 is very high for this superlinear model. While there may be other functions with superior goodness of fit, we are only seeking to determine the shape of the curve with reasonable accuracy.

7.10 For reference we attempted a 1st order linear regression and derived only 0.56 for the R^2. The linear model was significantly inferior to the superlinear model.

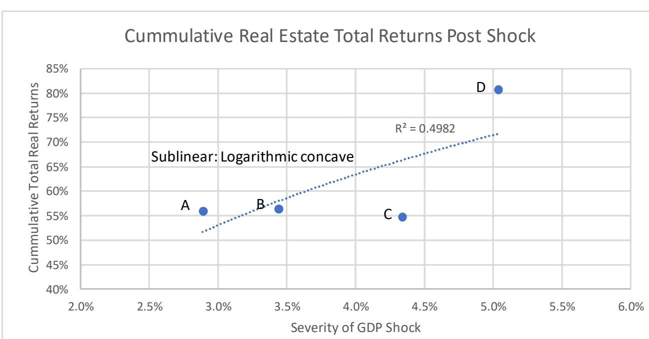
7.11 We then attempted a sublinear function which was the logarithmic regression. The slightly concave curve has an even lower R^2 of 0.4982.

7.12 Based on the empirical data, the superlinear function had a goodness of fit that was far superior than either a linear or concave function. Based on this result, as well as the Jensen inequality proof detailed earlier, we conclude that the performance of real estate in the longer run is convex with respect to the severity of the perturbation in the economy brought about by a systemic shock or Black Swan event.

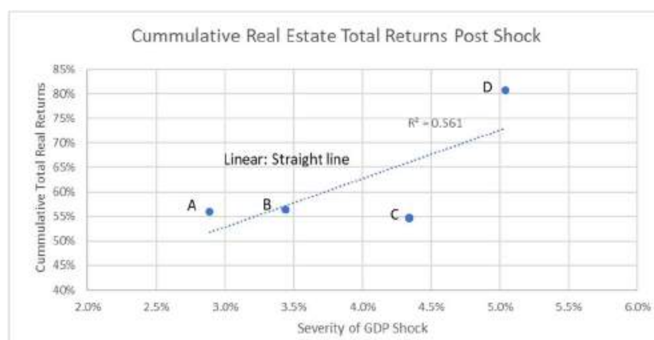
Superlinear Convex Model Best Fit: $f(x) = ax^2 + bx + c$



Sublinear Concave Model Best Fit: $f(x) = a.\ln(X) + b$



Linear Model Best Fit: $f(x) = ax + b$



8. Covid-19 Black Swan

8.1 As of this writing, the Covid-19 pandemic has resulted in over 2 million confirmed cases and 134,000 deaths based on data from John Hopkins[15]. With infections several order of magnitude greater than confirmed cases and estimates of case fatality ratios of 1·4% [0·4–3·5] in those aged <60 years and 4·5% [1·8–11·1] in those aged ≥60 years[14] numerous countries have instituted mandatory lock downs and stay at home policies. Estimates suggest deaths in the United States from Covid-19 ranging from 39,966 to 177,866[16]. Another study estimates deaths in the US of 2.2 million without non-pharmaceutical interventions and in the vicinity of 200,000 even with aggressive and recurrent suppression[13&17].

8.2 Following major pandemics, researchers have found, based on data from the 1314 to present that the real natural rate of interest tends to be depressed by 2% for decades following the pandemic[18]. The natural rate of interest is a real short-term rate that occurs when the economy has reached maximum employment and has stable inflation (i.e., the interest rate that occurs when the economy is in equilibrium)[19]. The reasoning is intuitive in that the psychology of individuals will be more cautionary post pandemics resulting in a greater concern for wealth preservation as opposed to return maximization.

8.3 The impact of Covid-19 will likely follow that of other pandemics / Black Swan events in history. If the real natural rate of interest manifests sustained, long run, declines as predicted by empirical data, yields from fixed income instruments will fall. Lower rates are constructive for real estate investment performance due to the impact on asset price reflation adding to the antifragility thesis of real estate.

Conclusion

9.1 Real estate as an asset class offers investors several key benefits when added to a multi-asset portfolio. Real estate's low volatility and low correlation relative to other asset classes, high proportion of total return attributable to current income versus appreciation, income secured by contractual leases from tenants are some of the key merits. In addition, real estate investments can be accessed via four quadrants of real estate, namely private equity, private debt, public equity and public debt. This provides a menu of risk and return alternatives for the investor.

9.2 Nassim Taleb's fourth quadrant scenario of fat-tailed risk with large consequence highlights frailty of multi-asset investment portfolios to systemic shocks. Black Swan events occur more frequently in the real world than predicated by traditional financial models. The implication is that multi-asset portfolios would benefit from assets that have convexity and antifragile characteristics. We posit based on both first principles and empirical data that private equity real estate investments have these desired attributes. Since private equity real estate underpins real estate debt (private and public) and public traded REITs, the antifragility benefits can be found in varying degrees in all 4 quadrants of real estate.

9.3 Current market turbulence brought about by the sudden impact of the coronavirus demonstrates vividly the impact of unknown unknowns on investment portfolios. Real estate's track record through other shocks, asymmetrically skewed right return distribution, and antifragility in the longer run suggests performance of multi-asset portfolios will benefit from higher allocations to real estate.

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